PHYS 393
Low Temperature Physics
Set 1:

Introduction and Liquid Helium-3

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Low Temperatures

- Low compared to what?
- Many definitions possible
- Coldest natural system: 2.7K (CMBR)
- Things happen when the interaction energy between the constituents of a material is of the order $k_B T$
  - 10K: superconductivity in metals
  - 2.17K: He^4 superfluidity
  - 2mK: He^3 superfluidity

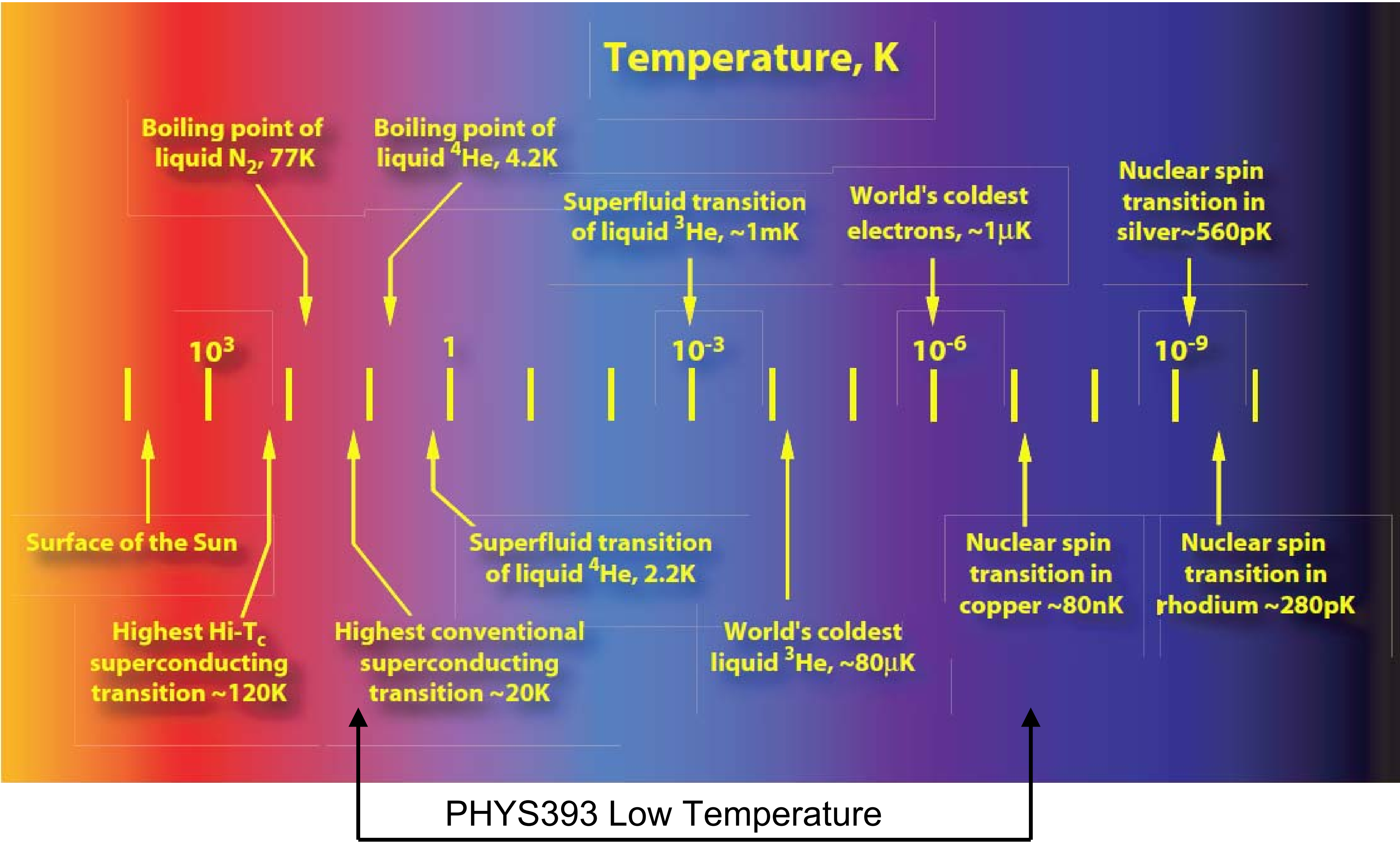
Low Temperatures

- Low Temperature effects:
  - Superconductivity
  - Superfluidity
- Strange – spectacular effects
- Macroscopic properties determined by quantum mechanics
- Rich area of experimentation for 100 years
- Source/motivation of tremendous insights and progress in theoretical physics
- LT-motivated theories often apply to many areas of physics, e.g. spontaneous symmetry breaking -> particle physics (Physics Nobel Prize 2008)
- Theoretical understanding still not complete

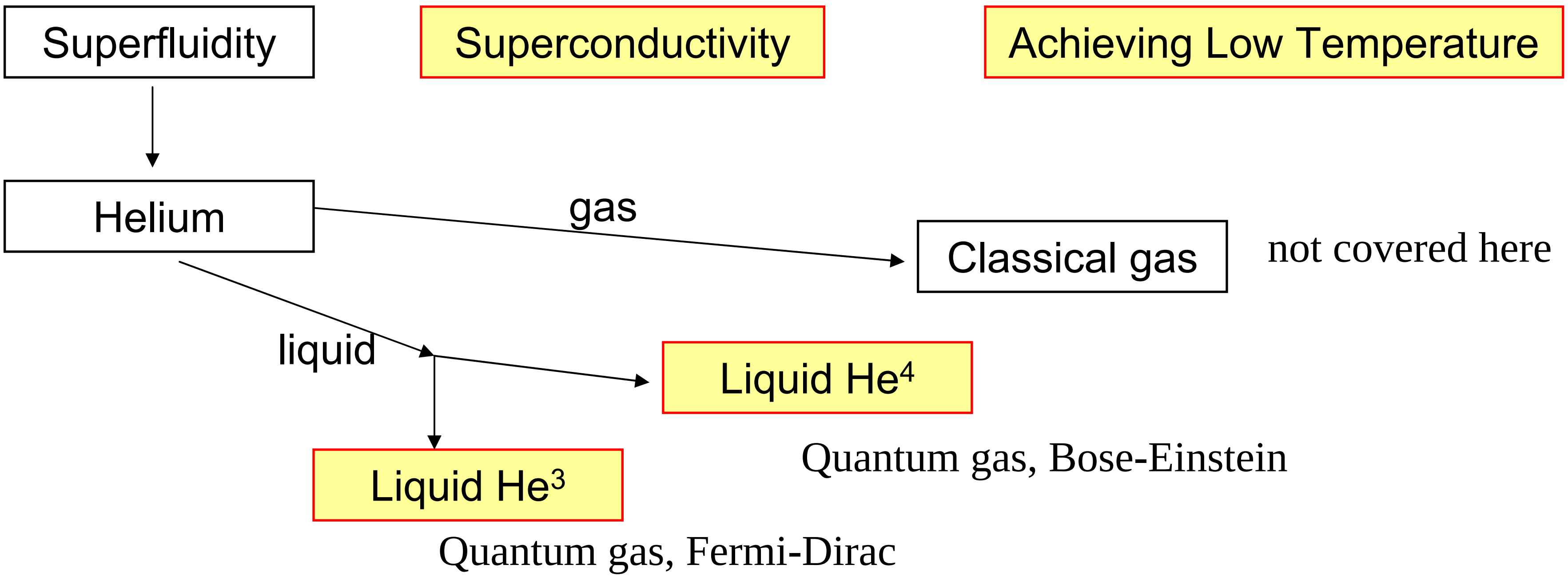
Low Temperatures

- Nobel Prizes:
  - 1913, 1962, 1972, 1973, 1977, 1978, 1987, 1996, 1997, 1998, 2001, 2003 (Physics)
  - 1949 (Chemistry)
- Applications: NMR, MRI, particle accelerators (Tevatron, LHC), fusion (ITER)
- LHC @ CERN:
  - Superconducting magnets
  - Cooled to 1.9K circulating superfluid Helium

Low Temperatures



Low Temperature Physics in PHYS 393



We don’t cover laser cooling and Bose-Einstein condensates

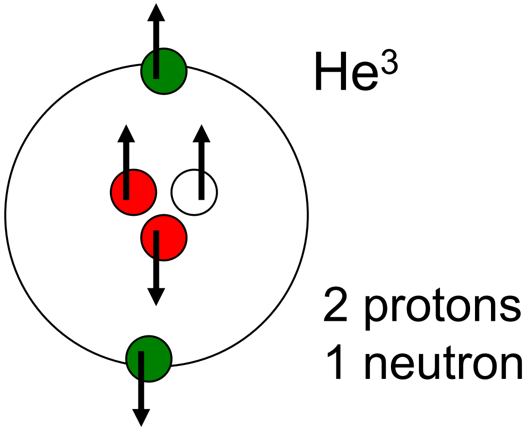
Helium

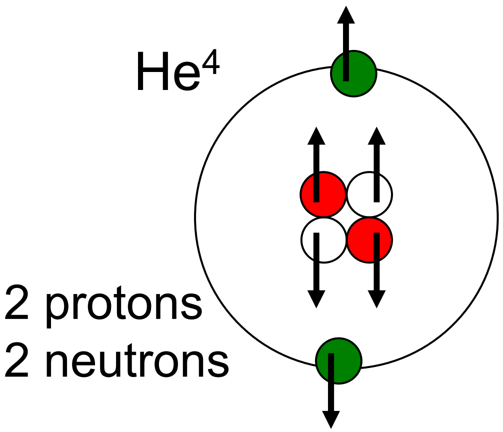
- He first observed in solar spectrum (1868)
- Discovered on Earth in 1895 in mineral containing U and Th
- Kamerlingh Onnes (Leiden) liquefied He in 1908: establishment of low-temperature physics
- He^4 found at 5.2ppm in atmosphere; not gravitationally bound; replenished by radioactive decay
- Extracted from natural gas
- He^3 discovered in 1933
- Produced through Li neutron capture (waste from nuclear reactors or hydrogen bomb materials)
- Liquefied in 1949

Helium atom

- Two electrons in K shell ($1s^2$)
- Spherical shape, no permanent electric dipole moment
- Smallest atomic polarizability
- Very weak diamagnetic susceptibility
- Smallest atom: r=31pm
- Highest ionization energy: 24.6eV

Helium atoms





He^3

2 protons

1 neutron $\quad I = \frac{1}{2}$

electrons$(1s)^2 \quad J = 0$

He^4

2 protons

2 neutrons $\quad I = 0$

He^3
F=1/2
Fermion

Total atom $\vec{F} = \vec{I} + \vec{J}$

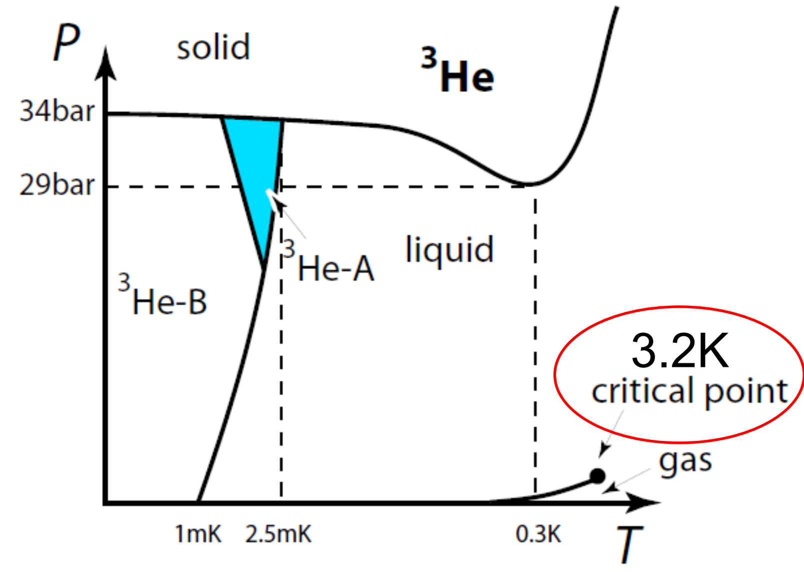
He^4
F=0
Boson

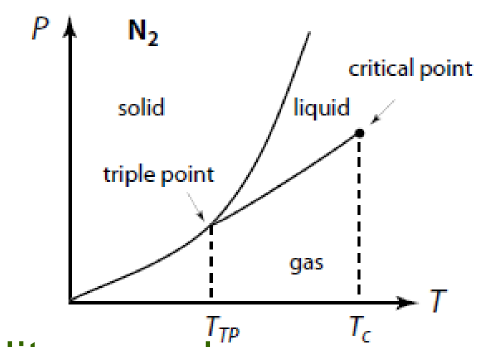
- He^4 and He^3 exhibit very different macroscopic properties
- This reflects mostly their different quantum nature (boson vs fermion)

Helium 3

- Normal liquid: 0.003K<T<3.2K
  - boiling point: 3.2K
- Superfluid: T<0.003K
- Phase diagram (state of He^3 as function of pressure and temperature): under atmospheric pressure liquid down to T=0. No triple point (where gas, liquid, solid co-exist)
- Near T=0 solidifies for P>33atm

Helium 3 phase diagram





Nitrogen phase diagram for comparison (note triple point)

Solid to T=0K

- Solid vs liquid: competition between attractive forces binding atoms and energy of individual atoms which reduces binding

- Van der Waals force: electric dipole-dipole interaction. He atoms have no permanent EDM, but quantum fluctuations induce fluctuating EDMs at neighbouring atoms leading to energy reduction (attractive force)

- Strength of VdW force proportional to atomic polarizability: small in He atoms, leading to very weak VdW forces

Solid to T=0K

- Zero Point Energy due to Heisenberg uncertainty
- Classical regime: $E=1/2(p^2/m)$
- Atom constrained in volume V, radius $R \sim V^{1/3}$
- Heisenberg: $\Delta p \sim \hbar/R$
- Small atomic mass m leads to large E_0

$$E_0 \sim \frac{\hbar^2}{2mV^{2/3}}$$

At normal pressure Z.P.E. is always larger than VdW binding energy, hence He^3 (and He^4) remain liquid even at T=0

Melting curve

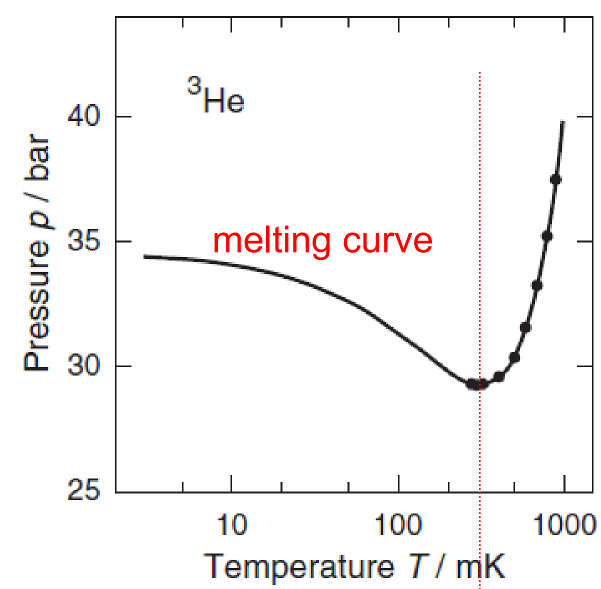
- Unique feature: melting curve has negative dP/dT in 3mK<T<0.32K
- Clausius-Clapeyron equation:

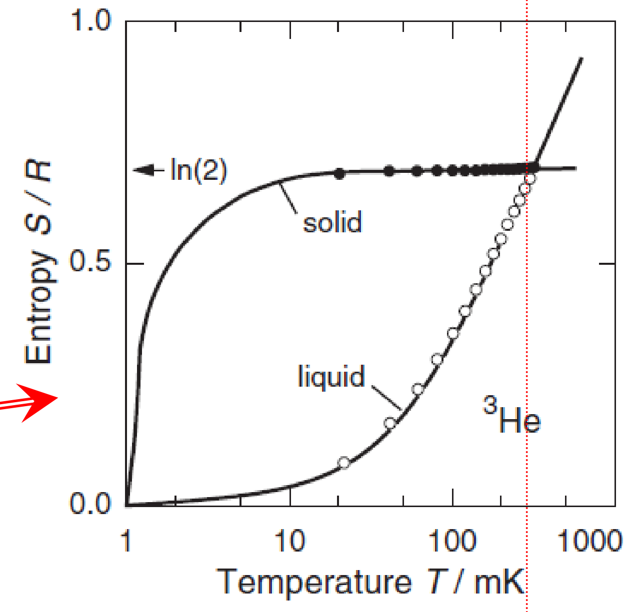
$$\left.\frac{\partial p}{\partial T}\right|_{\mathrm{melting curve}} = \frac{S_\ell - S_\mathrm{s}}{V_\ell - V_\mathrm{s}}$$

where ℓ, s denote liquid and solid phase, S is entropy, V is volume

- In most materials:
  - $S_l > S_s$, liquid more disordered than solid
  - $V_l > V_s$, liquid has lower density than solid
- Negative dP/dT can be due to:
  - $V_l < V_s$ (water), or
  - $S_l < S_s$ liquid He^3

phase diagram





Entropy of solid He^3

- In 3mK<T<0.32K entropy S dominated by disorder of nuclear spin (1/2)

- In solid we have N localized (hence distinguishable) spin ½ particles in 3 energy states separated by $\Delta\varepsilon$: Boltzman distribution (see Wolski part 2 page 14):

$$S = -\left(\frac{\partial F}{\partial T}\right)_V = Nk \ln\left(1 + e^{-\frac{\Delta\varepsilon}{kT}}\right) + Nk\frac{(\varepsilon/kT)e^{-\frac{\Delta\varepsilon}{kT}}}{\left(1 + e^{-\frac{\Delta\varepsilon}{kT}}\right)}$$

$$\Delta\varepsilon / k \simeq\sim 2mK \;\; ; \;\; \text{for } T \gg 2mK \;\; : \;\; S_S \to Nk \ln 2$$

Entropy of liquid He^3

- In 3mK<T<0.32K He^3 behaves like a degenerate Fermi gas (T_F=4.9K)
- Indistinguishable particles
- At low T all states below Fermi Energy filled with spin-up, spin-down pairs: no spin disorder
- Only particles with energy within kT of ε_F can flip spin and generate smal spin disorder
- For T<TF the heat capacity of a Fermi gas is given by (see Wolski part 4 page 48):

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{\pi^2}{2} Nk \frac{T}{T_F}$$

$$S_l = \int_0^T \frac{C_V}{T} dT, \quad C_V = \gamma T \quad \Rightarrow \quad S_l = \gamma T$$

Summary on liquid He^3 entropy

- For 3mK < T < 0.32K:

$$S_S = Nk\ln 2 > S_l = \gamma T$$

- This gives negative dP/dT melting curve

- Practical use: by applying high pressure to liquid it solidifies and due to entropy difference this leads to drop in temperature: Pomeranchuk cooling (discussed later)

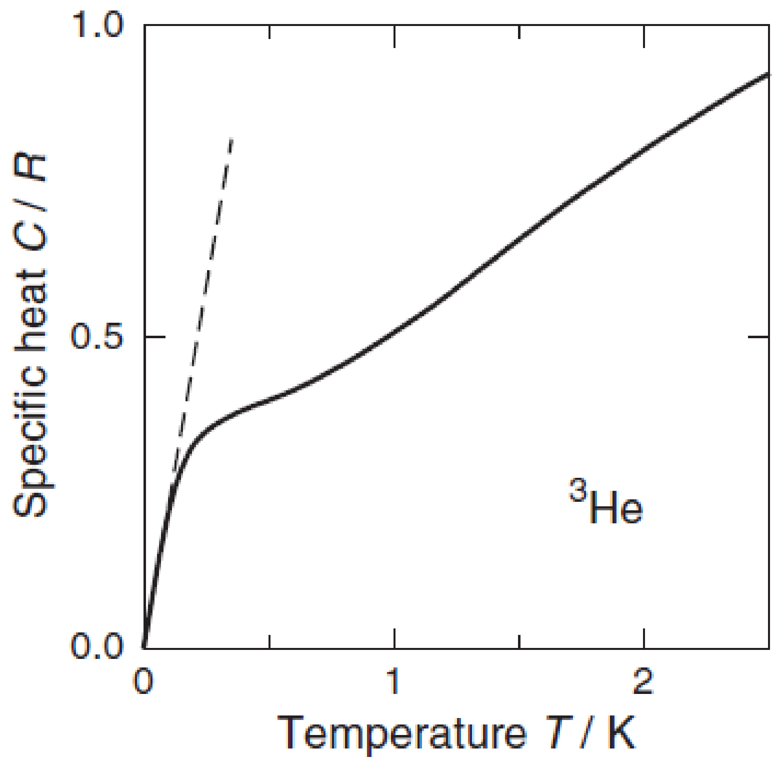
Heat Capacity

- Fermi gas (see previous):

$$C_V = \gamma T$$

- True only for T<0.2K

Normalized specific heat C/R of liquid ^3He at a molar volume of $36.82\,cm^3\,mol^{-1}$ as a function of temperature.



Thermal Conductivity Λ

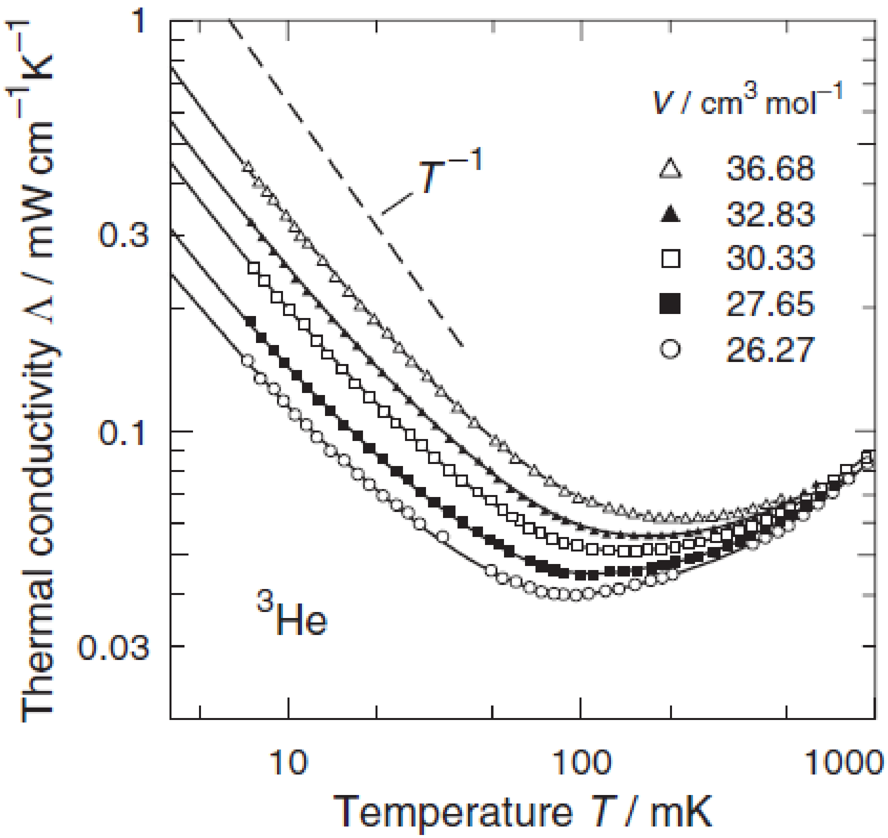
- Fermi gas:

  $\Lambda \sim 1/T$

- In the absence of external pressure this is a good approximation for very low T

- Liquid He^3 is very poor heat conductor (at 1K Λ is 50 times less than for glass)

Thermal conductivity Λ of liquid ^{3}He as a function of the temperatures at different molar volumes. At low temperatures and a molar volume of $36.68\,\mathrm{cm^3\,mol^{-1}}$ one finds roughly a $1/T$ dependence, as indicated by the *dashed line*



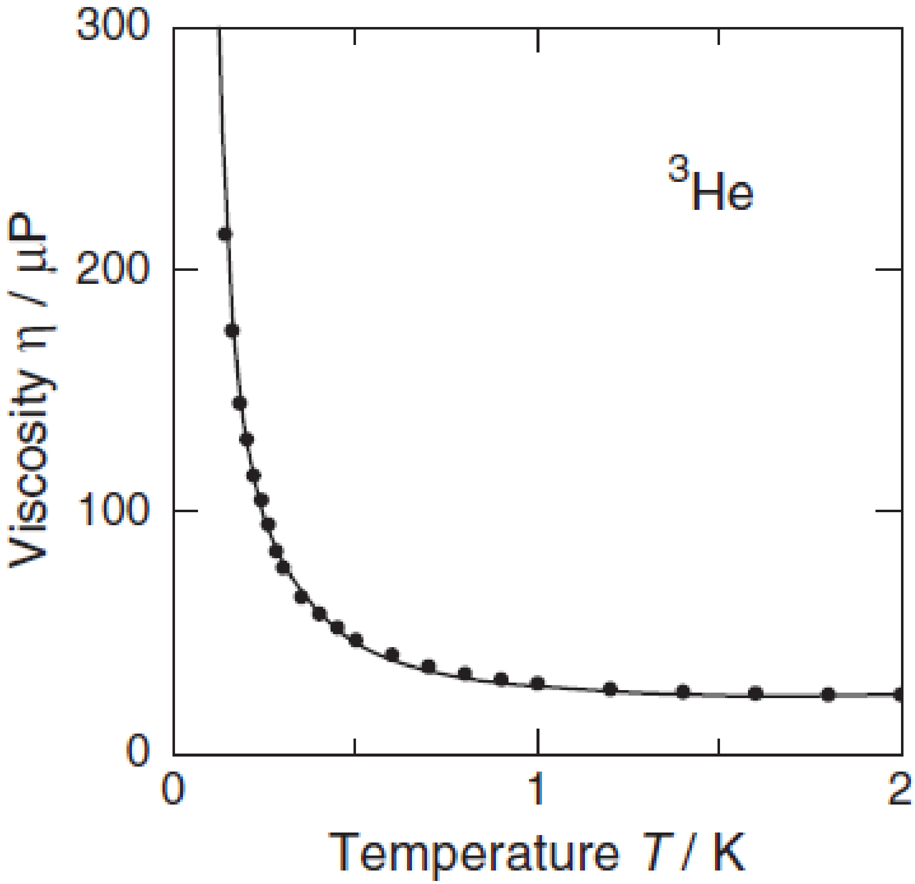
Viscosity

- Fermi gas:

  $$\eta \sim 1/T^2$$

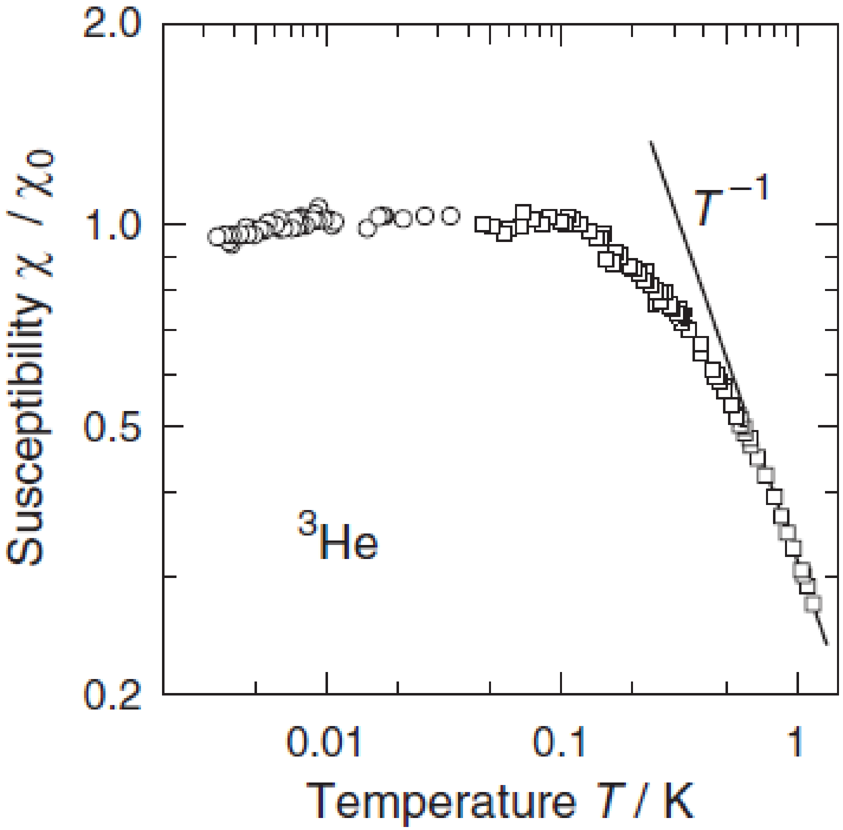
- Reasonable agreement up to T=2K

- Liquid He^3 at low T (just above transition temperature to superfluid state) has viscosity comparable to that of honey



Magnetic susceptibility

- Fermi gas predicts magnetic susceptibility independent of T (see Wolski part 4 page 64)
- Reasonable agreement for 3mK < T < 0.4K



Fermi Liquid Theory

- Fermi gas describes quantitatively He^3 properties at $T<0.2K$
- Landau improved agreement by taking into account the strong interaction between atoms: Fermi Liquid
- Instead of single atom excitations the model considers collective excitations: quasiparticles with different energies but unmodified momenta
- Quasiparticle is effectively particle+interaction
- Replace particle mass m with effective mass m*
- This modifies energy levels
- m* is 3 to 6 times the particle mass depending on measured quantity
- Much better description of measured values

Energy levels in Fermi gas (left) and Fermi liquid (right)